Stochastic Resonance under Two-Parameter Modulation in a Chemical Model System

Takashi Amemiya,* Takao Ohmori, Tetsuya Yamamoto,† and Tomohiko Yamaguchi*

Department of Chemical Systems, National Institute of Materials and Chemical Research (NIMC), 1-1 Higashi, Tsukuba, Ibaraki 305-8565, Japan

Received: December 17, 1998; In Final Form: March 3, 1999

Stochastic resonance (SR) is demonstrated under *two-parameter* modulation in a model of the photosensitive Belousov−Zhabotinsky reaction in a flow system. Light flux and a flow rate are two control parameters in the model, and modulate a focal steady state close to a Hopf bifurcation simultaneously. Addition of noise to the light flux gives rise to SR whose behavior depends on the periods of and phase difference in the two modulations. Compared with *one-parameter* modulation by the light flux, the additional flow rate modulation results in both increase and decrease in the resonance, strong resonance with a period different from that of a signal, or suppressed resonance. These computational results are interpreted by a threshold scenario of SR applied to the *two-parameter* system. The effect of the flow rate modulation on SR in the photosensitive system is also studied from the viewpoint of the peristaltic effect of a pump.

I. Introduction

In chemical reaction systems, both theoretical and experimental studies have revealed the effects of external fluctuations on the nonlinear dynamics. Periodic modulations by light,[1−3] flow rates,[4−6] or electrical current[7] have been found to induce entrainment,[1,2,4,6,8,9] resonance,[3,5,7] and chaos.[6−8] Stochastic perturbations applied to flow rates have lead to bursting oscillations in the methylene blue oscillator,[10] sustained oscillations in the glycolysis of yeast,[11] and convergence of chaos in a model of the Belousov−Zhabotinsky (BZ) reaction.[12,13]

Stochastic resonance (SR)[14] may occur when both periodic and stochastic fluctuations are applied to nonlinear systems. Examples of SR have been found in a wide range of physical,[15,16] chemical,[17−22] and biological[15,16] systems. The theory of SR was originally developed for a double-well potential function or a bistable system,[14,17,18,23] and the concept of SR has now been extended to include transitions between two different dynamical states.[15,16,19−21] In such dynamical systems, SR is similar to noise-induced phase transitions.[24−26] Kai and co-workers have recently proposed noise synchronization (NS)[27] for SR-like entrainment phenomena in the photosensitive BZ reaction[28−32] where optical noise crosses a threshold between oscillatory and steady states. We have extended the idea of SR to two-parameter systems[33−35] in the BZ reaction, where Hopf bifurcations have been used as thresholds. In the context of the two-parameter SR, we have considered a signal and noise that belong to different physical sources, e.g., light flux and a flow rate,[33,34] or temperature and a flow rate,[35] and demonstrated that SR may occur when a signal and noise are applied to the different inputs, respectively.

The present paper is a series of SR studies in the reported system[33] and demonstrates SR under two-parameter modulation; a system is exposed to a periodic signal and noise in one parameter and simultaneously to another periodic modulation in a different parameter. It is worth noting that there has been no report on SR under conditions where other periodic modulations than a signal are applied to a system, though many examples of SR have been reported in different scientific areas.[15−21,36,37]

In the present paper, we use the same model of the photosensitive BZ reaction in a flow system that was developed previously.[33] Light flux and a flow rate of solutions are experimentally controllable parameters and are independent bifurcation parameters in the model. First, a periodic signal and noise were applied to the light flux with no modulation in the flow rate. Second, periodic modulations with different periods and phases were applied to the flow rate under the same conditions of the signal and noise in the light flux. The SR behavior was compared in both cases. Thus, SR study in the present system can be regarded as either SR under two-parameter modulation or, more specifically, the effect of additional flow rate modulation on SR in the photosensitive system. From the second viewpoint, the peristaltic effect of a pump is also examined.

II. Model

Photosensitive Flow-Oregonator. The dimensionless form of the three-variable model[33] of the photosensitive BZ system is given by the following differential equations by using the Tyson's scaling of the Oregonator:[38,39]

$$\begin{Bmatrix} \epsilon\,\dot{x} \\ \epsilon'\,\dot{y} \\ \dot{z} \end{Bmatrix} = \begin{Bmatrix} x(1-x) + y(q-x) \\ 2hz - y(q+x) \\ x - z \end{Bmatrix} + \begin{Bmatrix} -\epsilon x \\ -\epsilon'(y-y_0) \\ -z \end{Bmatrix}\kappa_{\mathrm{f}} + \begin{Bmatrix} p_2 \\ p_1 \\ (p_1/2+p_2) \end{Bmatrix}\phi \quad (1)$$

or simply

$$\dot{\boldsymbol{X}} = \boldsymbol{A} + \boldsymbol{F}\kappa_{\mathrm{f}} + \boldsymbol{P}\phi \quad (2)$$

where $\dot{\boldsymbol{X}}$ consists of the production rate of the activator (x), inhibitor (y), and oxidized catalyst (z) with the scaling constants,

* To whom correspondence should be addressed.
† Permanent address: Department of Electrical Engineering, Tokyo Metropolitan College of Technology, Shinagawa, Tokyo 140-011, Japan.

10.1021/jp984786n CCC: $18.00
Published on Web 04/15/1999

ϵ and ϵ'; $\boldsymbol{A}$ is the kinetic term of the Oregonator model,[38] $\boldsymbol{F}\kappa_f$ is the flow term, $\boldsymbol{P}\phi$ is the photosensitive term; q is another scaling constant, h is the stoichiometric factor;[38] p_1 and p_2 are the coefficients of photosensitivity determined by concentrations of bromate, sulfuric acid, and bromomalonic acid.[33] The external control parameters are the light flux, ϕ, and the flow rate, κ_f. The linear stability analysis of eq 2 in the two-parameter space can be found in the literature[33] for two different compositions of the reactant solutions. The present study used one of the same solute compositions as that reported previously,[33] in which the concentration (0.05 M) of bromomalonic acid was higher. The period of the autonomous oscillations in this system was 60 s under dark and zero flow rate conditions and increased to 100 s by increasing the light flux and the flow rate.

For the SR study, a periodic signal and random noise were applied in the light flux and a periodic modulation was applied in the flow rate as follows:

$$\phi = \phi^0 \left(1 + \alpha_1 \sin\left(\frac{2\pi}{T_1} t - \varphi_1\right) + \beta_1 \xi(\delta)\right) \tag{3}$$

$$\kappa_f = \kappa_f^0 \left(1 + \alpha_2 \sin\left(\frac{2\pi}{T_2} t - \varphi_2\right)\right) \tag{4}$$

where ϕ^0 and κ_f^0 are the constant values that define a focal steady state of the system; α_1 and α_2 are the amplitude of the sinusoidal signal and the modulation with periods T_1 and T_2, respectively; φ_1 and φ_2 are the initial phases of the signal and modulation; β_1 is the noise amplitude of equally distributed random numbers, $\xi(\delta)$, between -1 and 1; and δ is the duration (5 s) of the pulse of a single noise event that occurred every 10 s.[19-21,33]

Evaluation of SR. To quantify the SR effect, we analyzed the time series (30 000 s) of the data by examining an interspike distribution at a period of interest, which is an alternative to examining the signal-to-noise ratio from the power spectra of the corresponding time series.[40] Here, we considered a normalized number of adjacent spikes (NNAS) to see the SR effect,

$$\mathrm{NNAS} = \frac{\mathrm{NAS}^{T}{}_{\mathrm{signal}}}{\mathrm{TN}_{\mathrm{signal}}} - \frac{\mathrm{NAS}^{T}{}_{\mathrm{noise}}}{\mathrm{TN}_{\mathrm{noise}}} \tag{5}$$

where NAS $^{T}{}_{\mathrm{signal}}$ is the number of adjacent spikes (NAS)[33] whose time interval was in the range between $(1 - 0.1)T$ and $(1 + 0.1)T$ (T is the period of interest) in the case of a signal and noise in the light flux with or without a periodic modulation in the flow rate; $\mathrm{TN}_{\mathrm{signal}}$ is the total number of spikes under the same conditions as above; $\mathrm{NAS}^{T}{}_{\mathrm{noise}}$ is the number of adjacent spikes (NAS) whose time interval was in the range between $(1 - 0.1)T$ and $(1 + 0.1)T$ in the case of only noise in the light flux with no periodic modulation in the flow rate; $\mathrm{TN}_{\mathrm{noise}}$ is the total number of spikes under the above conditions. The definition of eq 5 is a modified version of the NAS as reported previously[33] and is a more reliable indication for quantifying SR in the present system since NNAS takes into account the background conditions, i.e., only noise in the system.[41]

III. Results

The effect of additional flow rate modulation on SR in the case of a signal and noise in the light flux is shown in Figure 1. The period of the flow rate modulation (300 s) was the same as that of the signal, while the initial phase was changed. When an in-phase ($\varphi_1 = \varphi_2 = 0$) periodic modulation was added to the flow rate, significant enhancement in the SR effect was observed compared with the case of no modulation in the flow rate. Contrarily, when an out-of-phase ($\varphi_1 = 0, \varphi_2 = \pi$) modulation was added to the flow rate, the SR curve became flat and exhibited no characteristic feature, indicating the suppression of the resonance. These results demonstrate that the SR behavior in the photosensitive system is changed dramatically depending on the difference in the phase between the flow rate modulation and the signal in the light flux.

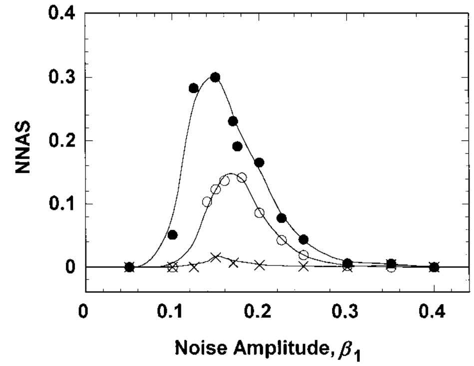

Figure 1. Interspike distribution in the time series (30 000 s) of oscillations in the model system (eqs 2−4). Normalized number of adjacent spikes (NNAS) in y whose time interval was between 270 and 330 s, i.e., $(1 - 0.1)T_1$ and $(1 + 0.1)T_1$, was plotted as a function of a noise amplitude (β_1) in the light flux. Equations 2−4 were solved numerically using the Gear method with constant values of a parameter's set, $h = 0.5$, $\epsilon = 4.29 \times 10^{-1}$, $\epsilon' = 2.32 \times 10^{-3}$, $q = 9.52 \times 10^{-5}$, $y_0 = 4.76 \times 10$, $p_1 = 1.12 \times 10^{-1}$, $p_2 = 6.89 \times 10^{-1}$, $\phi^0 = 9.94 \times 10^{-4}$, $\kappa_f^0 = 1.05 \times 10^{-3}$, under the three different perturbed conditions: (○) signal and noise in the light flux without flow rate modulation, $T_1 = 300$ s, $\alpha_1 = 0.032$, $\varphi_1 = 0$, $\alpha_2 = 0$; (●) signal and noise in the light flux under in-phase flow rate modulation, $T_1 = 300$ s, $\alpha_1 = 0.032$, $\varphi_1 = 0$, $T_2 = 300$ s, $\alpha_2 = 0.038$, $\varphi_2 = 0$; (×) signal and noise in the light flux under out-of-phase flow rate modulation, $T_1 = 300$ s, $\alpha_1 = 0.032$, $\varphi_1 = 0$, $T_2 = 300$ s, $\alpha_2 = 0.038$, $\varphi_2 = \pi$. The focal steady state is located 13.3% above the Hopf bifurcation of the flow rate and 10.6% of the light flux[33] according to Schneider's notation.[19-21] A different random number was used in each calculation, and the NNAS obtained from three calculations was averaged at each noise amplitude. The standard deviations of the NNAS were reasonable to quantify the SR effect.[33] The solid lines are to guide the reader's eye.

We have next investigated the effect of the flow rate modulation on the SR behavior by changing both the period and the initial phase. The added flow rate modulation had a smaller period ($T_2 = 200$ s) than that ($T_1 = 300$ s) of a signal in the light flux, and the two initial phases were different ($\varphi_1 = 0$, $\varphi_2 = 2\pi/3$). Without the flow rate modulation (Figure 2a), SR with the period ($T_1 = 300$ s) of the signal was observed as expected. We also examined SR with the period of 200 and 600 s to compare with the results under the additional flow rate modulation; a smaller characteristic peak for SR was observed for 600 s, and a little characteristic feature for SR was observed for 200 s as also shown in Figure 2a. When the flow rate modulation was applied, significant increase in the SR was observed with the period of 600 s, which was neither the period of the signal nor the period of the flow rate modulation. In contrast, SR with the signal itself almost disappeared, and SR with the flow rate modulation increased instead. The above results indicate that a system may exhibit strong SR with a period different from that of a signal itself when the system is exposed to another modulation than the signal.

We have finally studied the effect of the flow rate modulation from the standpoint of the peristaltic effect of a pump. The period of the flow rate modulation was set to 1 s since the usual peristaltic periods are in the range 0.1−20 s,[42] which are much smaller than the periods of the signal and the autonomous

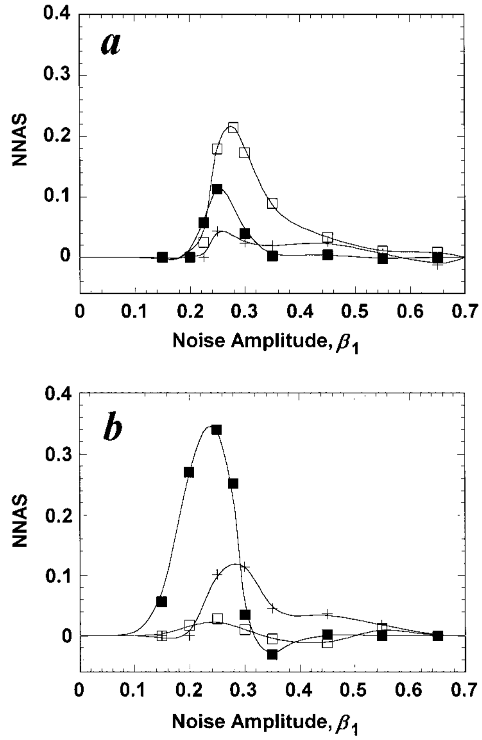

Figure 2. Interspike distribution obtained in the same constant parameters' set as shown in Figure 1 except for a focal steady state ($\phi^0 = 9.94 \times 10^{-4}$, $\kappa_f^0 = 1.2 \times 10^{-3}$) and under different perturbed conditions: (a) signal and noise in the light flux without flow rate modulation, $T_1 = 300$ s, $\alpha_1 = 0.068$, $\varphi_1 = 0$, $\alpha_2 = 0$; (b) signal and noise in the light flux under flow rate modulation, $T_1 = 300$ s, $\alpha_1 = 0.068$, $\varphi_1 = 0$, $T_2 = 200$ s, $\alpha_2 = 0.08$, $\varphi_2 = 2\pi/3$. The NNAS whose time interval was 300 ± 30 s (□), 200 ± 20 s (×), and 600 ± 60 s (■). The focal steady state is located 29.4% above the Hopf bifurcation of the flow rate and 28.0% of the light flux. The solid lines are to guide the reader's eye.

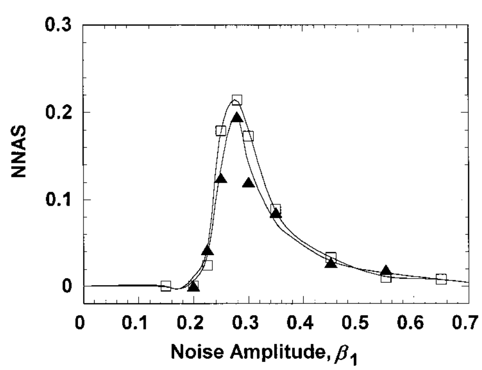

Figure 3. Interspike distribution obtained in the same constant parameters' set as shown in Figure 2 under different perturbed conditions, i.e., the peristaltic pump modulation: (□) signal and noise in the light flux without flow rate modulation, $T_1 = 300$ s, $\alpha_1 = 0.068$, $\varphi_1 = 0$, $\alpha_2 = 0$; (▲) signal and noise in the light flux under a peristaltic pump modulation, $T_1 = 300$ s, $\alpha_1 = 0.068$, $\varphi_1 = 0$, $T_2 = 1$ s, $\alpha_2 = 0.08$, $\varphi_2 = 0$. The solid lines are to guide the reader's eye.

oscillations. The results are shown in Figure 3, demonstrating little effect of a peristaltic pump on the SR. Calculations were also carried out with different initial phases of the flow rate modulation, which gave SR curves virtually the same as that shown in Figure 3.

IV. Discussion

The results of SR obtained in this study, i.e., both increase and decrease in the resonance, strong resonance with a period

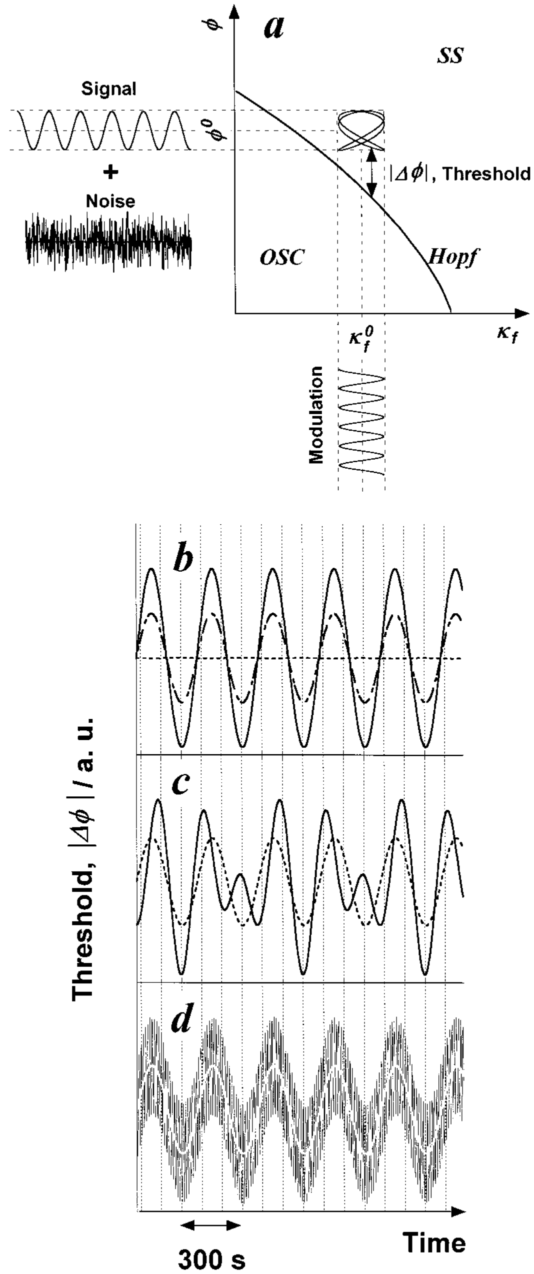

Figure 4. (a) Schematic representation of the threshold change under two-parameter modulation in the space of light flux (ϕ) and a flow rate (κ_f). A Hopf bifurcation line (Hopf) divides the region into the steady-state (SS) and the oscillatory state (OSC). A trajectory of a focal steady state under two-parameter modulation is shown as a lissajou's figure. The threshold of light flux is indicated by $|\Delta\phi|$. (b)−(d) The change in the threshold in time under different flow rate modulations: (b) in-phase (—) and out-of-phase (- - -) flow rate modulation, and no flow rate modulation (— - —) as shown in Figure 1; (c) a small period (200 s) flow rate modulation (—) and no flow rate modulation (- - -) as shown in Figure 2; (d) a peristaltic pump modulation (—) and no flow rate modulation (white line) as shown in Figure 3. The threshold change was calculated by assuming that the Hopf bifurcation curve[33] is linear in the small perturbation range.

different from that of a signal, and the suppression of the resonance, can be interpreted in the framework of the threshold scenario[43] of SR under *two-parameter* modulation. Both a signal in the light flux and a periodic modulation in the flow rate perturb a focal steady state and consequently change the threshold as shown in Figure 4. When an out-of-phase flow rate modulation with the same period of a signal is added to the focal point, the change in the threshold becomes negligible, resulting in the suppression of the resonance; the trajectory of the focal point as shown in Figure 4a becomes almost parallel to the Hopf bifurcation line. Contrarily, the in-phase flow rate modulation increases the change in the threshold value (Figure 4b) and results in the enhancement of the SR. A modified periodic change in the threshold can be induced as shown in Figure 4c, when a different flow rate modulation is applied to the system. In this case, the minimum of the threshold value appears at a period of 600 s, and SR with this period becomes significant, while SR with the signal almost disappears.

The flow rate modulation with a period of a few seconds, which is the case of using a peristaltic pump, also induces the change in the threshold in a similar manner as above as shown in Figure 4d. Though the threshold is seemed to change noisy because of the flow rate modulation with much smaller periods than that of the signal, the periodicity of the signal is totally maintained. Further, the amplitude of the peristaltic modulation used in this SR study was less than 10% of the constant value, which is known to have little effect on the oscillatory behavior of the Oregonator model as well.[4,42] Therefore, the flow rate modulation with a period of 1 s is considered to have little effect on the SR behavior. This result is also understandable if we note that SR cannot occur with a signal whose period is smaller than that of the autonomous oscillations in the present system.[19−21] More detailed examination should be necessary to account for the small decrease in the SR effect under the peristaltic flow rate modulation (Figure 3).

V. Conclusion

We have shown characteristic behavior of SR such as increase and decrease in the resonance, strong resonance with a period different from that of a signal, and the suppression of the resonance under two-parameter modulation in the photosensitive BZ system. The idea of two-parameter modulation studied in the chemical system can be applied to another model such as the FitzHugh−Nagumo (FHN) neuron model,[44,45] and as well as to experiments for neurophysiological sensory systems[40,46] and ion channels.[47]

Stochastic resonance under two-parameter or in general multiparameter modulation may widely be seen in biological and natural systems, because environmental fluctuations are inherent in nature[15,16] and any living and nonliving systems are exposed to many kinds of periodic and stochastic fluctuations simultaneously.[48−50] In such a case, biological systems may enhance a signal by using another periodic modulation from different external sources or by adjusting their own internal rhythms to the signal.

Acknowledgment. We are grateful to Professor S. Kai of Kyushu University for valuable comments and to Drs. K. Kato, K. Seki, and R. Iseki of NIMC for discussion. We thank Research Information Processing Station, Agency of Industrial Science and Technology, MITI, for their support on the numerical calculation of the ordinary differential equations by CRAY C90.

References and Notes

(1) Dulos, E.; De Kepper, P. *Biophys. Chem.* **1983**, *18*, 211.
(2) Steinbock, O.; Zykov, V.; Müller, S. C. *Nature* **1993**, *366*, 322.
(3) Petrov, V.; Ouyang, Q.; Swinney, H. L. *Nature* **1997**, *388*, 655.
(4) Markman, G.; Bar-Eli, K. *J. Phys. Chem.* **1994**, *98*, 12248.
(5) Buchholtz, F.; Schneider, F. W. *J. Am. Chem. Soc.* **1983**, *105*, 7450.
(6) Markus, M.; Müller, S. C.; Hess, B. *Ber. Bunsen-Ges. Phys. Chem.* **1985**, *89*, 651.
(7) Förster, A.; Zeyer, K.-P.; Schneider, F. W. *J. Phys. Chem.* **1995**, *99*, 11889.
(8) Kai, T.; Tomita, K. *Prog. Theor. Phys.* **1979**, *61*, 54.
(9) Rehmus, P.; Ross, J. *J. Chem. Phys.* **1983**, *78*, 3747.
(10) Resch, P.; Münster, A. F.; Schneider, F. W. *J. Phys. Chem.* **1991**, *95*, 6270.
(11) Boiteux, A.; Goldbeter, A.; Hess, B. *Proc. Natl. Acad. Sci.* **1975**, *72*, 3829.
(12) Matsumoto, K.; Tsuda, I. *J. Stat. Phys.* **1983**, *31*, 87.
(13) Yoshimoto, M.; Yamaguchi, T.; Sato, H.; Kato, K.; Mahara, H.; Nagashima, H. *J. Phys. Soc. Jpn.* **1998**, *67*, 103.
(14) Benzi, R.; Sutera, A.; Vulpiani, A. *J. Phys. A* **1981**, *14*, L453.
(15) Moss, F., Bulsara, A., Shlesinger, M. F., Eds. *Proc. NATO Adv. Res. Workshop Stochastic Reson. Phys. Biology*, *J. Stat. Phys.* **1993**, *70*, 1.
(16) Mannella, R., McClintock, P. V. E., Eds. *Proc. Int. Workshop Fluctuations Phys. Biology: Stochastic Reson., Signal Processing Related Phenomena, Nuovo Cimento* **1995**, *17D*, 1.
(17) Leonard, D. S.; Reichl, L. E. *Phys. Rev. E* **1994**, *49*, 1734.
(18) Dykman, M. I.; Horita, T.; Ross, J. *J. Chem. Phys.* **1995**, *103*, 966.
(19) Guderian, A.; Dechert, G.; Zeyer, K.-P.; Schneider, F. W. *J. Phys. Chem.* **1996**, *100*, 4437.
(20) Förster, A.; Merget, M.; Schneider, F. W. *J. Phys. Chem.* **1996**, *100*, 4442.
(21) Hohmann, W.; Müller, J.; Schneider, F. W. *J. Phys. Chem.* **1996**, *100*, 5388.
(22) Kádár, S.; Wang, J.; Showalter, K. *Nature* **1998**, *391*, 770.
(23) Wiesenfeld, K.; Moss, F. *Nature* **1995**, *373*, 33.
(24) Kai, S.; Kai, T.; Takata, M.; Hirakawa, K. *J. Phys. Soc. Jpn.* **1979**, *47*, 1379.
(25) Horsthemke, W.; Lefever, R. *Noise-Induced Transitions: Theory and Applications in Physics, Chemistry and Biology*; Springer-Verlag: Berlin, 1984.
(26) Brand, H. R.; Kai, S.; Wakabayashi, S. *Phys. Rev. Lett.* **1985**, *54*, 555.
(27) Fujii, K.; Inomoto, O.; Kai, S. *Technol. Rep. Kyushu University* **1998**, *72*, 1 (in Japanese).
(28) Krug, H.-J.; Pohlmann, L.; Kuhnert, L. *J. Phys. Chem.* **1990**, *94*, 4862.
(29) Jinguji, M.; Ishihara, M.; Nakazawa, T. *J. Phys. Chem.* **1992**, *96*, 4279.
(30) Yamaguchi, T.; Shimamoto, Y.; Amemiya, T.; Yoshimoto, M.; Ohmori, T.; Nakaiwa, M.; Akiya, T.; Sato, M.; Matsumura-I, T. *Chem. Phys. Lett.* **1996**, *259*, 219.
(31) Kaminaga, A.; Mori, Y.; Hanazaki, I. *Chem. Phys. Lett.* **1997**, *279*, 339.
(32) Kádár, S.; Amemiya, T.; Showalter, K. *J. Phys. Chem. A* **1997**, *101*, 8200.
(33) Amemiya, T.; Ohmori, T.; Nakaiwa, M.; Yamaguchi, T. *J. Phys. Chem. A* **1998**, *102*, 4537.
(34) Amemiya, T.; Ohmori, T.; Nakaiwa, M.; Yamaguchi, T. *ACH−Models Chem.* **1998**, *135*, 231.
(35) Amemiya, T.; Ohmori, T.; Nakaiwa, M.; Yamamoto, T.; Yamaguchi, T. *J. Biol. Phys.*, in press.
(36) Barzykin, A. V.; Seki, K.; Shibata, F. *Phys. Rev. E* **1988**, *57*, 6555.
(37) Yamazaki, H.; Yamada, T.; Kai, S. *Phys. Rev. Lett.* **1998**, *81*, 4112.
(38) Field, R. J.; Noyes, R. M. *J. Chem. Phys.* **1974**, *60*, 1877.
(39) Tyson, J. J. *Ann. N.Y. Acad. Sci.* **1979**, *316*, 279.
(40) Douglass, J. K.; Wilkens, L.; Pantazelou, E.; Moss, F. *Nature* **1993**, *365*, 337.
(41) We could get reliable SR curves by subtracting the NAS^{T}_{noise} from the NAS^{T}_{signal}. It was not suitable to divide the NAS^{T}_{signal} by the NAS^{T}_{noise} in the present system, because no spikes were generated until the noise amplitudes became relatively large under the background conditions. Division of the each NAS by each TN was also necessary to take into account the different perturbed conditions.
(42) Bar-Eli, K. *J. Phys. Chem.* **1985**, *89*, 2852.
(43) Jung, P. *Phys. Rev. E* **1994**, *50*, 2513.
(44) FitzHugh, R. *Biophys. J.* **1961**, *1*, 445.
(45) Nagumo, J. S.; Arimoto, S.; Yoshizawa, S. *Proc. IRE* **1962**, *50*, 2061.
(46) Gluckman, B. J.; Netoff, T. I.; Neel, E. J.; Ditto, W. L.; Spano, M. L.; Schiff, S. J. *Phys. Rev. Lett.* **1996**, *77*, 4098.
(47) Bezrukov, S. M.; Vodyanoy, I. *Nature* **1995**, *378*, 362.
(48) Hastings, M. *Nature* **1995**, *376*, 296.
(49) Grenfell, B. T.; Wilson, K.; Finkenstädt, B. F.; Coulson, T. N.; Murray, S.; Albon, S. D.; Pemberton, J. M.; Clutton-Brock, T. H.; Crawley, M. J. *Nature* **1998**, *394*, 674.
(50) Leirs, H.; Stenseth, N. C.; Nichols, J. D.; Hines, J. E.; Verhagen, R.; Verheyen, W. *Nature* **1998**, *398*, 176.